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TO THE PROBLEM OF $K_L \to \pi^0\gamma\gamma$ DECAY

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It is shown that the contribution of intermediate vector mesons to the $K_L \to \pi^0\gamma\gamma$ amplitude, intensively discussed in literature in recent years, is close to zero provided that the group SU(3) breaking is taken into account. At the same time, the contribution of intermediate scalar mesons is essential. The obtained estimates for $Br(K_L \to \pi^0\gamma\gamma)$ conform with the recent experimental data $Br(K_L \to \pi^0\gamma\gamma) = (2.1 \pm 0.6)\cdot 10^{-6}$.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

О проблеме $K_L \to \pi^0\gamma\gamma$ распада

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В работе показано, что вклад промежуточных векторных мезонов в амплитуду распада $K_L \to \pi^0\gamma\gamma$, интенсивно обсуждавшийся в литературе последних лет, близок к нулю при учете нарушения группы SU(3). В то же время заметную роль играет вклад от промежуточных скалярных мезонов. Полученные оценки на $Br(K_L \to \pi^0\gamma\gamma)$ удовлетворяют существующим экспериментальным данным $Br(K_L \to \pi^0\gamma\gamma) = (2,1 \pm 0,6)\cdot 10^{-6}$.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Recently, the $K_L \to \pi^0 e^+ e^-$ decay is intensively discussed in literature[1-5]. An increased interest in this decay stems from a possibility of studying on its basis the nature of CP parity violation in future experiments $K_L \to \pi^0 e^+ e^-$ [6]*. If the CP parity is violated, the decay proceeds through a one-photon intermediate state. However, a competing process conserving the CP parity is possible here which proceeds through a two-photon intermediate state, $K_L \to \pi^0\gamma\gamma \to \pi^0 e^+ e^-$. Therefore, for a thorough study of the $K_L \to \pi^0 e^+ e^-$ decay we need good information on the $K_L(p) \to \pi^0(p_1)\gamma(q_1)\gamma(q_2)$ process too. The amplitude of the latter can be written as follows:

**Note that as early as 1966 it was proposed for the first time in [7] to verify the CP invariance by studying the $K_L \to \pi^0 e^+ e^-$ decay.*

$$T_{K_L \to \pi^\circ \gamma\gamma} = \epsilon_\mu(q_1)\epsilon_\nu(q_2)\{A(q_1^\nu q_2^\mu - g^{\nu\mu} q_1 q_2) + B[-(pq_1)(pq_2)\, g^{\mu\nu} - (q_1 q_2)\, p^\mu p^\nu + (pq_1)\, p^\nu q_2^\mu + (pq_2)\, p^\mu q_1^\nu]\}. \tag{1}$$

The $K_L \to \pi^\circ e^+e^-$ decay can essentially be influenced by the part of the amplitude (1) containing factor B. The first term of (1) gives the contribution to the $K_L \to \pi^\circ e^+e^-$ decay, proportional to the electron mass, and therefore, it can be neglected.

Factor B in the amplitude (1) is determined by the contributions of diagrams with intermediate vector mesons (ρ and ω, see fig.1) [1-3]. In this paper we show that if the SU(3) group breaking is taken into account [8], the contribution of these diagrams to the sum of three transitions $K_L \to (\pi^\circ, \eta, \eta') \to \pi^\circ\gamma\gamma$ is almost equal to zero in the region of most probable values of mixing angles of singlet-octet components of η mesons, $-20^\circ \leq \theta \leq -18^\circ$. Thus, it turns out that the competing role of the CP conserving part of the amplitude is small and the one-photon intermediate state in this reaction should play the decisive role, which makes it easier to observe the CP violation.

At the end of this paper we shall show that factor A is influenced not only by the meson-loop contribution $K_L \to \pi^\circ\pi^+\pi^- \to \pi^\circ\gamma\gamma$ discussed earlier [1, 2, 9] but also by the transitions $K_L \to (\pi^\circ, \eta, \eta') \to \pi^\circ\gamma\gamma$ mediated by scalar mesons $f_0(700)$, $f_0(975)$, $f_0(1400)$ and $a_0(983)$ (see fig.2) [10, 11].

In order to describe the $K_L \to (\pi^\circ, \eta, \eta')$ transitions we take the effective Lagrangian of weak interactions in the form [12, 13, 8]

$$L_F^{eff} = G_F/\sqrt{2}\, s_1 c_1 c_3 Q_{\Delta S=1}, \tag{2}$$

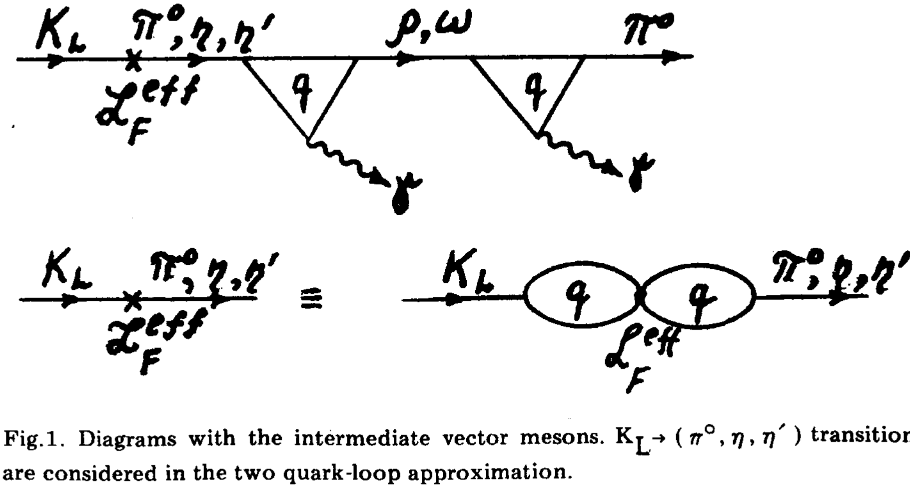

Fig.1. Diagrams with the intermediate vector mesons. $K_L \to (\pi^\circ, \eta, \eta')$ transitions are considered in the two quark-loop approximation.

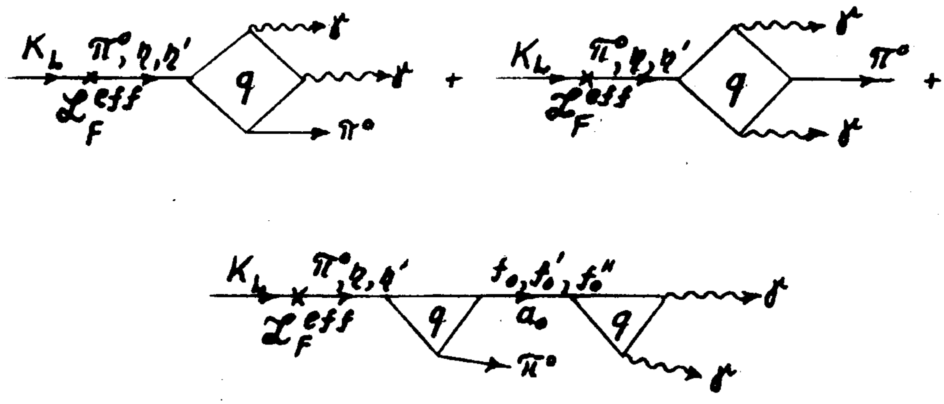

Fig.2. Contact diagrams with the intermediate scalar mesons.

where

$$Q_{\Delta S=1} = Q = Q_1 - 1.6Q_2 + 0.033Q_3 - 0.018Q_5 + 0.1Q_6 ,$$

$G_F/\sqrt{2} s_1 c_1 c_3 = 1.77 \cdot 10^{-6}$ GeV^{-2}, $s_i = \sin\phi_i$, $c_i = \cos\phi_i$ are elements of the Kobayashi-Maskawa matrix [14] and Q_i are four-quark operators. As an example, we give here only the most important operator Q_6 (the operator of the "penguin"-type)

$$Q_6 = [\bar{s}_a \gamma^\nu (1-\gamma^5) d_b] \sum_{q=u,d,s} [\bar{q}_b \gamma_\nu (1+\gamma^5) q_a] .$$

Here a, b = 1, 2, 3 are colour indices. The remaining operators can be found in ref. [8, 12]. The effective Lagrangian (2) satisfies the selection rules $|\Delta S| = 1$, $|\Delta I| = 1/2, 3/2$. Since the value of the matrix elements of the transitions $\langle K_L | Q | \pi^0, \eta, \eta' \rangle$ depends mainly on the operator Q_6, we write down these elements in the explicit form only for the operator mentioned above[8]

$$\langle \pi^0 | Q_6 | K^0 \rangle = \rho X ,$$

$$\langle \eta | Q_6 | K^0 \rangle = [-(2/3 + \rho) \sin\theta' + \sqrt{2}\, F_s / F_\pi (1/3 + \rho') \cos\theta'] X , \quad (3)$$

$$\langle \eta' | Q_6 | K^0 \rangle = [-(2/3 + \rho) \cos\theta' - \sqrt{2}\, F_s / F_\pi (1/3 + \rho') \sin\theta'] X .$$

Here $\theta' = \theta_0 - \theta$, where $\theta_0 = 35.26^\circ$ is the angle of ideal mixing and

$\theta = -18°$ or $-20°$, $F_\pi = 93$ MeV, $F_K = 1.2F_\pi$ and $F_S = 1.3F_\pi$ are the decay constants of π, K mesons and a pseudoscalar state involving only strange quarks [15]. $X = \langle \pi^o | Q_I | K^o \rangle = 3.5 \cdot 10^{-3}$ (GeV)^4. The parameters ρ and ρ' are equal to

$$\rho = 64(1+\lambda)\,(Z m_u F_\pi / M F_K)^2 \,[1 - \lambda F_K^2/(2(1+\lambda)F_\pi^2)] \cong 47,$$

$$\rho' = 64\lambda(1+\lambda)\,(Z m_u F_\pi^2 / M F_K F_S)^2\,[1 - F_K^2/(2(1+\lambda)F_\pi^2)] \cong 61, \qquad (4)$$

where $\lambda = m_s/m_u = 1.64$, $Z^{-1} = 0.5\{1 + [1 - (2g_\rho F_\pi/m_{a_1})^2]^{1/2}\}$ is the renormalisation constant of 0^- mesons, caused by the transitions $0^- \to 1^+$ (1^+-axial-vector meson), $m_{a_1} = 1260$ MeV is the mass of the axial-vector meson a_1 [17], $m_u = m_{a_1}[(Z-1)/6Z]^{1/2} = 280$ MeV is the u quark mass, m_s is the s quark mass [15, 16] *, g_ρ is the constant of the decay $\rho \to 2\pi$ ($g_\rho^2/4\pi = \alpha_\rho \cong 3$) and M is the K meson mass.

It is seen from the above formulae that in the case of exact SU(3) symmetry ($m_u = m_s$, $F_\pi = F_K = F_S$) (3) and (4) result in a usual SU(3) symmetric relation between the matrix elements $\langle K^o | Q_6 | \pi^o, \eta, \eta' \rangle$ which have been used in papers [1-3].

Using formulae, given in paper [8], one can obtain the following values for the matrix elements of the transitions $K^o \to \pi^o, \eta, \eta'$ for two different values of the angle θ

$$\theta = -18° \quad \langle \pi^o|Q|K^o\rangle = 4.9X; \quad \langle \eta|Q|K^o\rangle \doteq 3X; \quad \langle \eta'|Q|K^o\rangle = -10.6X. \qquad (5)$$

$$\theta = -20° \quad \langle \pi^o|Q|K^o\rangle = 4.9X; \quad \langle \eta|Q|K^o\rangle = 2.6X; \quad \langle \eta'|Q|K^o\rangle \doteq -10.7X;$$

For the decay amplitude $K_L \to \gamma\gamma$ we have [8] ($\alpha = 1/137$)

$$T_{K_L \to \gamma\gamma} = \frac{\alpha G_F s_1 c_1 c_3}{3\pi F_\pi}\left\{ \frac{3\langle \pi^o|Q|K^o\rangle}{M^2 - m_\pi^2} + (5\sin\theta' - \sqrt{2}\cos\theta' F_\pi/F_s) \times \right.$$

$$\left. \times \frac{\langle \eta|Q|K^o\rangle}{M^2 - m_\eta^2} + (5\cos\theta' + \sqrt{2}\sin\theta' F_\pi/F_s)\frac{\langle \eta'|Q|K^o\rangle}{M^2 - m_{\eta'}^2} \right\} =$$

$$= \begin{cases} 4.5 \cdot 10^{-9}\ \text{GeV}^{-1}, & \theta = -18° \\ 3.4 \cdot 10^{-9}\ \text{GeV}^{-1}, & \theta = -20°. \end{cases}$$

**Mass* $m_u = 280$ *MeV corresponds to the value* $m_{a_1} = 1260$ *MeV* [17]. *If* $m_{a_1} = 2g_\rho F_\pi = 1140$ *MeV, then* $m_u = 330$ *MeV. The experimental value of* m_{a_1} *has not yet been established definitely and is within the limits mentioned here* [16,18]. *Formulae (3) and (4) have been derived in the approximation of two quark-loops providing transitions* $K^o \to \pi^o, \eta, \eta'$ *(see* [8] *and Fig.1).*

The experimental values are equal to [17]

$\Gamma_{K_L \to \gamma\gamma} = (7.24 \pm 0.35) \cdot 10^{-12}$ eV, $\quad T_{K_L \to \gamma\gamma} = 3.4 \cdot 10^{-9}$ GeV $^{-1}$

It is seen that at $\theta = -20°$ one can obtain a good agreement between theoretical and experimental data.

$K_L \to \pi^\circ\gamma\gamma$ a) Vector Mesons

The contributions of diagrams with intermediate vector mesons (Fig.1) to factor B (see (1)) equal

$$B = \frac{5\alpha\alpha_\rho G_F s_1 c_1 c_3 \langle \pi^\circ | Q | K^\circ \rangle}{\pi^2 F_\pi^2 [(p-q_1)^2 - m_\rho^2](M^2 - m_\pi^2)} \Delta , \tag{6}$$

where $(m_\rho = m_\omega)$

$$\Delta = 1 + \frac{3\sin\theta' \langle \eta | Q | K^\circ \rangle (M^2 - m_\pi^2)}{5 \langle \pi^\circ | Q | K^\circ \rangle (M^2 - m_\eta^2)} + \frac{3\cos\theta' \langle \eta' | Q | K^\circ \rangle (M^2 - m_\pi^2)}{5 \langle \pi^\circ | Q | K^\circ \rangle (M^2 - m_{\eta'}^2)} , \tag{7}$$

In the case of exact SU(3) symmetry, using formulae given in papers [1,3], for the coefficient Δ we get*

$\Delta = 1 - 0.09 + 0.21 = 1.1 \quad (\theta = -18°)$,

$\Delta = 1 + 0.03 + 0.21 = 1.24 \quad (\theta = -20°)$.

However, if the breaking of the SU(3) group is taken into account this coefficient sharply decreases due to the compensating influence of the η meson pole. Indeed, formulae (5) and (7) result in

$\Delta = 1 - 1.270 + 0.266 = -0.04 \quad (\theta = -18°)$,

$\Delta = 1 - 1.13 + 0.26 = 0.13 \quad (\theta = -20°)$.

**Analogous results can be obtained from formulae (3) and (4) if $F_\pi = F_K = F_S$, $\lambda = 1$, $\rho = \rho'$. Here, the contribution of the η meson is very small and at $\theta = -19.5°$ it equals zero because $\langle \eta | Q_6 | K^\circ \rangle = 3^{1/2} \rho X \cos(2\theta_0 - \theta)$.*

Thus, in the region of most probable values of the mixing angle, $-20° \le \theta \le -18°$, the coefficient Δ is very small and runs through the zero value under the change of θ in this interval. The amplitude (6) will give a negligible contribution to the general width of the decay $K_L \to \pi^\circ\gamma\gamma$

$$\Gamma^{(\rho,\omega)}_{K_L\to\pi^\circ\gamma\gamma} = 10^{-18}\,\mathrm{eV} \quad (\theta = -18°)\,,$$
$$\Gamma^{(\rho,\omega)}_{K_L\to\pi^\circ\gamma\gamma} = 7\cdot 10^{-16}\,\mathrm{eV} \quad (\theta = -20°) \tag{8}$$

that can be verified by comparing (8) with the recent experimental result [19]

$$\Gamma_{K_L\to\pi^\circ\gamma\gamma} = (2.7\pm 0.8)\cdot 10^{-14}\,\mathrm{eV}\,, \quad Br(K_L\to\pi^\circ\gamma\gamma) = (2.1\pm 0.6)\cdot 10^{-6}. \tag{9}$$

b) Scalar Mesons

To describe the part of the amplitude $K_L \to (\pi^\circ, \eta, \eta') \to \pi^\circ\gamma\gamma$ which is connected with the processes proceeding through intermediate scalar mesons and contact vertices (see Fig.2), one can use the results obtained in [10a,b] in which the decay $\eta\to\pi^\circ\gamma\gamma$ width and pion polarizability were calculated. Then for the factor A (see (1)) we get

$$A^{(\square, a_0, f_0)} = \frac{10\alpha G_F s_1 c_1 c_3 \langle\pi^\circ|Q|K^\circ\rangle}{9\pi F_\pi^2 (M^2 - m_\pi^2)}\Delta'\,, \tag{10}$$

where

$$\Delta' = \frac{3}{5}\Big[\frac{\sin\theta'\langle\eta|Q|K^\circ\rangle(M^2-m_\pi^2)}{\langle\pi^\circ|Q|K^\circ\rangle(M^2-m_\eta^2)} + \frac{\cos\theta'\langle\eta'|Q|K^\circ\rangle(M^2-m_\pi^2)}{\langle\pi^\circ|Q|K^\circ\rangle(M^2-m_{\eta'}^2)}\Big]\Big[1-\frac{4m_u^2}{m_{a_0}^2-s}\Big] +$$
$$+ 1 - 4m_u^2\Big[\frac{1}{m_{f_0}^2 - s - i\sqrt{s}\,\Gamma_{f_0}(\sqrt{s})\,\theta(s-4m_\pi^2)} + \frac{c}{m_{f'}^2 - s} + \frac{1}{m_{f''_0}^2 - s}\Big]$$
$$(s = (q_1+q_2)^2\,).$$

Here, in the channels with mesons η and η' there appears an isovector scalar meson $a_0 I^-(0^{++})$ with mass $m_{a_0} = 983\,\mathrm{MeV}$; and in the channel with π° meson, three isoscalar scalar resonances $0^+(0^{++})$, two of which $f'_0(975)$ and $f''_0(1400)$ are well known and are contained in the tables of experimental data [17] and $f_0(700)$ is not yet uniquely determined though in many experimental papers there are indications of its existence (see [20]). In the linear σ-model the scalar meson $f_0(700)$ plays the role of a σ particle, the lightest isoscalar state [15,21,22]. It should be taken into account in describing within the σ-model such processes as π–π scattering, pion polarizability, etc.[10b, 15, 22]. This meson has the mass in the range 700-900 MeV and a large decay width into two pions, which makes its detection very difficult. Therefore, in formula (10) one should take into account its width[15] $(m_\sigma^2 \simeq m_\pi^2 + 4m_u^2)$

$$\Gamma_{f_0}(m_{f_0}) = (3Z/2\pi m_{f_0})(m_u^2/F_\pi)^2[1-(2m_\pi/m_{f_0})^2]^{1/2} . \qquad (11)$$

The remaining scalar mesons have considerably smaller decay widths. Therefore, the width will be taken into account only for the $f_0(700)$ meson.

Note also that the $f_0'(975)$ meson mainly consists of strange quarks and only a small admixture of u and d quarks favours its decay into two pions with a relatively small width (26 MeV)[17]. With allowance for experimental data on the decay of this meson into two photons [23]

$$\Gamma_{f_0 \to \gamma\gamma} = (0.24 \pm 0.06 \pm 0.15)\ \text{keV (MARK II)}; = (0.31 \pm 0.14 \pm 0.09)\ \text{(Cryst.Ball)}.$$

one can introduce the coefficient $c = 0.07$ in formula (10) and to establish that the influence of this resonance on the decay $K_L \to \pi^0\gamma\gamma$ is insignificant. Thus, in further calculations it can be neglected.

The contributions of contact diagrams in the amplitude (10) almost cancel out and scalar mesons give decisive contributions to the total decay width, $K_L \to \pi^0\gamma\gamma$, commensurable only with the contribution of the channel $K_L \to \pi^0\pi^+\pi^- \to \pi^0\gamma\gamma$ [2,9]

$$\Gamma^{(\square,\, a_0,\, f_\sigma f_0'')}_{K_L \to \pi^0\gamma\gamma} = \begin{cases} 1.3 \cdot 10^{-14}\ \text{eV} & \theta = -18^\circ \\ 0.9 \cdot 10^{-14}\ \text{eV} & \theta = -20^\circ \end{cases} \qquad m_{f_0} = 730\ \text{MeV}\ ^{[8,10,15]}.$$

c) Mesons Loops

Following paper [2] we give expressions for factor A of the $K_L \to \pi^0\pi^+\pi^- \to \pi^0\gamma\gamma$ and $K_L \to \pi^0 K^+K^- \to \pi^0\gamma\gamma$ amplitudes

$$A^{(\pi,K)} = 5.1\,\alpha G_F s_1 c_1 c_3/\sqrt{2}\pi[(1-m_\pi^2/s)F(s/m_\pi^2) - (1-(M^2+m_\pi^2)/s)F(s/M^2)], \qquad (12)$$

where the loop function F(z) can be found in[2]. The amplitude (12) leads to the decay width

$$\Gamma^{(\pi,K)}_{K_L\to\pi^\circ\gamma\gamma} = 0.86\cdot 10^{-14}\,\mathrm{eV}.$$

The total contribution of the amplitudes (10) and (12) to the $K_L\to\pi^\circ\gamma\gamma$ width are (with allowance for the width of $f_0(730)$)

$$\Gamma_{K_L\to\pi^\circ\gamma\gamma} = \begin{cases} 3.9\cdot 10^{-14}\,\mathrm{eV} \\ 3.3\cdot 10^{-14}\,\mathrm{eV} \end{cases};\quad Br(K_L\to\pi^\circ\gamma\gamma) = \begin{cases} 3.07\cdot 10^{-6} & \theta=-18^\circ \\ 2.64\cdot 10^{-6} & \theta=-20^\circ \end{cases}$$

$m_{f_0} = 730$ MeV.

If for the mass of f_0 one takes the value 800 MeV we obtain

$$\Gamma_{K_L\to\pi^\circ\gamma\gamma} = \begin{cases} 3.0\cdot 10^{-14}\,\mathrm{eV} \\ 2.5\cdot 10^{-14}\,\mathrm{eV} \end{cases};\quad Br(K_L\to\pi^\circ\gamma\gamma) = \begin{cases} 2.3\cdot 10^{-6} & \theta=-18^\circ \\ 2.0\cdot 10^{-6} & \theta=-20^\circ \end{cases}$$

$m_{f_0} = 800$ MeV.

Finally, if the meson f_0 is removed from (10), we get too low values for the $K_L\to\pi^\circ\gamma\gamma$ width

$$\Gamma_{K_L\to\pi^\circ\gamma\gamma} = \begin{cases} 0.75\cdot 10^{-14}\,\mathrm{eV} & \theta=-18^\circ \\ 0.8\cdot 10^{-14}\,\mathrm{eV} & \theta=-20^\circ. \end{cases}$$

This fact may be considered one more important indication to the existence of a light scalar resonance with a large width.

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